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DYNAMIC COMPACTION MODEL FOR A GRANULAR MEDIUM

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A model for dynamic compaction of granular medium is proposed for the case in which the external action far exceeds the yield strength of the material. A radial axisymmetric compaction problem is solved for a granular medium with nanosize structure in the presence of a rigid rod at the symmetry axis. Simulated data are compared with experimental data on magnetic pulsed compaction of oxide nanopowders.

Key words: *powder compaction, plasticity, magnetic pulsed compaction.*

Recently, considerable efforts have been made to develop methods for producing new materials, in particular, nanoceramics. One promising method is the magnetic pulsed compaction of nanopowders with a conducting piston [1, 2]; particular cases of these method are the z-pinch technique, which involves electrodynamic compaction of a powder in a pipe being compressed by a pulsed magnetic field produced by a self-current and the θ-pinch technique, i. e., the radial induction compression of metal shells. In the description of compaction processes for structurally nonuniform bodies, considerable progress has been achieved within the framework of elasticity theory. A closed system of rheological equations for a plastic-strain-hardened porous body enabling consideration of various quasi-static processes has been formulated [3–5]. A serious limitation here is the assumption that, throughout the process, the stained state of the deformed body is on the loading surface. If the external loads grow rapidly, in particular, in experiments with strong pulsed magnetic fields used to compact a nanopowdered material, the latter condition doe not hold. In fast magnetic pulsed compaction processes, the inertial properties of the powder–shell system should be taken into account. Generally, to describe these phenomena adequately, it is necessary to integrate partial differential equations for the wave motions excited in the medium [5–7]. In solving the problem in question, one should take into account the (generally unknown) viscoelastic properties of the material being compacted in the limiting state. Because the wave nature of the motion of the medium is weakly manifested during compaction [1, 2], it can be assumed as a first approximation that the distribution of the final density in the compact is uniform and the characteristic rates of the processes involved are much lower than the sound velocity. Therefore, in the present paper, the inertia-related effects in the dynamic compaction problem for granular media are allowed for using the uniform compaction model.

We consider the problem of radial axisymmetric compaction of a granular medium in the presence of a rigid rod on the symmetry axis. We assume that longitudinal displacements are absent and that the elastic strains in the materials of the granules and shell are negligible compared to the plastic strain. The medium starts flowing when the external pressure reaches a certain threshold value determined by the yield strength of the granular medium and the copper shell. The maximum pressure $p_{\rm el}$ compensated by the elastic stress in the granular medium corresponds to a quasi-static compaction process. In a uniform compaction approximation, the quantity $p_{\rm el}$ is given by [4]

$$p_{\rm el} = \sqrt{\frac{1-\theta}{2}}\,\frac{2\Psi + \varphi/3 + \varphi/x^2}{\sqrt{2\Psi + \varphi/3 + \varphi/x^4}}\,\tau_0(\Gamma_0),$$

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where,

$$\Psi = \frac{2}{3}\,\frac{1-\theta+\theta^2/2}{\theta}\,\varphi(\theta), \qquad \varphi = (1-\theta)^{5/3}, \qquad x = \frac{R}{r_m},$$

$\theta = 1 - \rho/\rho_{\max}$ is the current porosity, ρ is the density of the medium, $\rho_{\max}$ is the density of the granular material, R is the outer radius of the medium compacted, and r_m is the rod radius. The yield stress τ_0 versus the shape-change strain measure Γ_0 can be calculated from experimental compression adiabats [2] for uniaxial compaction of a powder in a rigid press die. For instance, for an AM nanopowder [2] deformed at a strain rate of order 10^3–10^4 sec^{-1}, we obtain

$$\tau_0 = k_1\Gamma_0^{0.5} + k_2\Gamma_0^6/(0.014 + \Gamma_0^{5.5}), \tag{1}$$

where $k_1 = 0.530$ GPa and $k_2 = 2.005$ GPa. The hardening law for the shell material can be deduced from experimental uniaxial-loading data. For instance, the relation for copper similar to (1) is

$$\tau_c = K\sqrt{2/3}\sqrt{0.01 + \sqrt{2/3}\,\Gamma_c} \tag{2}$$

($K = 0.5415$ GPa is the strain-hardening coefficient) [8]. In the case of plastic flow, the shell velocity ($v_0 \simeq$ 100 m/sec) is much lower than the sound velocity; therefore, at each time, the shell exhibits a static distribution of elastic stresses. Using the solution of the classical Lamé problem [9] and assuming the material to be incompressible (Poisson's ratio $\sigma_p = 1/2$), for the intensity of the elastic-stress deviator, we obtain

$$\tau(r) = p_{c,\mathrm{el}} R_c^2 R^2 \sqrt{2}/[r^2(R_c^2 - R^2)], \tag{3}$$

where $p_{c,\mathrm{el}} = p(R_c) - p(R)$ is the difference between the external and internal pressures. It follows from (3) that the maximum stress intensity is attained at the inner wall of the shell ($r = R$). Equating $\tau(R)$ to the copper yield stress (2), we obtain the pressure difference, which, for plastic flow, is compensated by the elastic stress

$$p_{c,\mathrm{el}} = 2^{-1/2}\tau_c(1 - R^2/R_c^2).$$

In solving the equations of motion, we use cylindrical coordinates. Assuming that the shell material is incompressible, from the continuity equation we find the radial distribution of the velocity and the law of variation of the shell coordinates:

$$v(r) = v_0(t)R(t)/r, \qquad r = \sqrt{r_0^2 + R^2 - R_0^2}. \tag{4}$$

Here $v_0 = dR/dt < 0$ is the velocity of the inner surface and r_0 and R_0 are the initial values of r and R, respectively. Using (4), we find the intensity of the strain-rate tensor deviator in the shell:

$$\gamma_c(r) = \sqrt{2}\,|v_0|R/r^2.$$

According to (3), the plastic flow of the shell is due to the excess of the stress over the yield stress on the inner surface of the shell, where the shape-change strain measure is given by

$$\Gamma_c(R) = \int_0^t \gamma_c(R)\,dt = \sqrt{2}\,\ln\frac{R_0}{R}.$$

Then, with relations (4) taken into account, the equation of motion for the shell is an ordinary differential equation which defines the hydrostatic-pressure distribution $p(r)$. Since the plastic flow of the shell is due to the excess of the pressure load over the yield pressure $p_{c,\mathrm{el}}$, as the boundary condition on the outer surface we use the relation $p(R_c) = p_c - p_{c,\mathrm{el}}$ (p_c is the pressure load). As a result, the pressure exerted by the copper shell on the compacted powder is given by

$$p = p_c - p_{c,\mathrm{el}} + \rho_c(a_0 R + v_0^2)\ln(R_c/R) - (\rho_c/2)v_0^2(1 - R^2/R_c^2), \tag{5}$$

where ρ_c is the density of copper and $a_0 = dv_0/dt$ is the radial acceleration of the shell.

For the powder compacted, the radial velocity distribution and the dependence of the outer radius R on the porosity for the uniform compaction model are given by

$$v(r) = \frac{e}{2}\Big(r - \frac{r_m^2}{r}\Big), \qquad R(t) = \sqrt{r_m^2 + (R_0^2 - r_m^2)\,\frac{1-\theta_0}{1-\theta}} \tag{6}$$

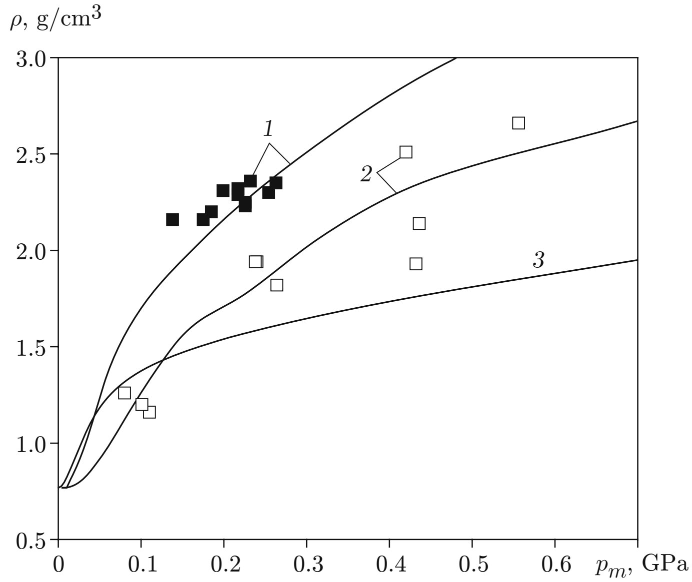

Fig. 1. Final compact density versus pressure load amplitude for $R_0 \simeq 10$ mm and $R_{c,0} - R_0 \simeq 1$ mm: the points refer to experiment, and the curves refer to calculation: 1) θ-pinch; 2) z-pinch; 3) calculation using a quasi-static approximation [4].

with $e = 2v_0/(R - r_m^2/R)$. The shape-change strain rate for the porous bed γ_0 is defined by the flow surface [4]. Then, using relations (6), for the shape-change strain measure we obtain

$$\Gamma_0(r) = \int\limits_0^t |\gamma_0|\, dt = \int\limits_\theta^{\theta_0} \sqrt{\Psi + \frac{\varphi}{6}\Big(1 + 3\,\frac{r_m^4}{r^4}\Big)}\,\frac{d\theta}{(1-\theta)^{3/2}}. \tag{7}$$

The dependence of Γ_0 on the radius can be one of the causes of nonuniform compaction of granular media in the presence of a rigid rod. It follows from (7) that this nonuniformity is located in the immediate vicinity of the rod ($r \sim r_m$). We ignore the dependence of Γ_0 on the radius and calculate the yield stress of the medium under compaction (1) using the value of Γ_0 that corresponds to the outer radius of the powder, i.e., $\Gamma_0(R)$. Using the condition of coaxiality of the deviators and the scalar implication of the associated law, for the principal components of the stress tensor in the powder layer, we obtain

$$\sigma_2 = \sigma_1\,\frac{3e\Psi - \varphi(v_r - 2v/r)}{3e\Psi + \varphi(2v_r - v/r)}, \qquad \sigma_3 = \frac{\sigma_2 v_r - \sigma_1 v/r}{v_r - v/r} \qquad \Big(v_r = \frac{\partial v}{\partial r}\Big).$$

Then, with the known velocity field (6) taken into account, the equation of motion for the powder becomes

$$\frac{\partial\sigma_1}{\partial r} + \frac{\sigma_1 - \sigma_2}{r} = \rho\,\frac{dv}{dt} = \rho a_0\,\frac{R}{r}\,\frac{r^2 - r_m^2}{R^2 - r_m^2} + \rho v_0^2\,\frac{r_m^2}{r^3}\,\frac{(r^2 - r_m^2)(R^2 - r^2)}{(R^2 - r_m^2)^2}. \tag{8}$$

Equation (8) is an ordinary differential equation that defines the distribution of the radial strain σ_1 in a specimen under compaction. As the boundary conditions, one should use the relations $\sigma_1(r_m) = p_{\rm el}(r_m) - p_*$ and $\sigma_1(R) = p_{\rm el}(R) - p$. We note that, generally, because the yield stress τ_0 increases with increasing strain rate [5], the pressure p_* exerted by the powder on the internal rod can exceed the yield load $p_{\rm el}(r_m)$. However, if the employed hardening law corresponds to the dynamic yield stress, it must be assumed that $p_* = p_{\rm el}(r_m)$. Then, taking into account the pressure p exerted on the powder, which is defined by relation (5), from (8) we obtain

$$\frac{Ra_0}{6\Psi + \varphi}\,A = -\frac{2}{\rho}\,\frac{(p_c - p_{c,\rm el} - p_{\rm el})x^2}{(6\Psi + \varphi)x^2 + 3\varphi} - v_0^2\Big(\frac{B}{(x^2-1)^2} + \frac{\rho_c}{\rho}\,\frac{2\ln(x_c/x) - 1 + x^2/x_c^2}{6\Psi + \varphi + 3\varphi/x^2}\Big), \tag{9}$$

where

$$A = 1 - \frac{6\Psi + 4\varphi}{(x^2-1)(6\Psi+\varphi)}\ln\Big(\frac{(6\Psi+\varphi)x^2 + 3\varphi}{6\Psi + 4\varphi}\Big) + \frac{\rho_c}{\rho}\,\frac{2x^2\ln(x_c/x)(6\Psi+\varphi)}{(6\Psi+\varphi)x^2 + 3\varphi},$$

$$B = \frac{(6\Psi + 4\varphi)[(6\Psi + \varphi)x^2 + 3\varphi]}{3\varphi(6\Psi + \varphi)^2} \ln\left(\frac{(6\Psi + \varphi)x^2 + 3\varphi}{6\Psi + 4\varphi}\right) - \frac{2x^2}{3\varphi} \ln(x) - \frac{x^2 - 1}{6\Psi + \varphi},$$

$x_c = R_c/r_m$. Relation (9), which is a nonlinear equation, governs the motion of the powder–shell interface. During magnetic pulsed compaction of a powder, the pressure on the conducting shell due to the external magnetic field varies in proportion to squared circuit current: $p_c \sim I^2$. In the case of a z-pinch, the time sweep of the current is a decaying harmonic signal [2], therefore, and in the calculations using Eq. (9) and ignoring decay effects, the loading-pulse waveform was specified as $p_c = p_m \sin^2(\pi t/T)$, where p_m is the pressure pulse amplitude and $T = 15$ μsec is the period. In the case of a θ-pinch, the current pulse waveform is distorted and the dependence $p_c(t)$ was specified by a more complex model signal: $p_c = p_m \sin^2(\pi t/(2t_0))$ for $0 \leqslant t < t_0$, $p_c = p_m$ for $t_0 \leqslant t < 3t_0$, and $p_c = 0$ for $t \geqslant 3t_0$ ($t_0 = 20$ μsec). A comparison of the theoretical and experimental data [2] on the final density of compacts of AM nanopowder is presented in Fig. 1. The same figure shows a curve of the quasi-static solution [4]. Good agreement between the calculated and experimental data is evident.

Thus, in the present study, the problem of dynamic compaction of a granular medium using the uniform compaction model is reduced to a second-order ordinary differential equation that governs the motion of the powder–shell interface, which considerably simplifies the calculations and analysis of the process. It is shown that, by taking into account the inertial properties of the conducting shell and the medium being compacted, one can achieve good agreement between theoretical data on the final density of compacts and experimental data on magnetic pulsed compaction of powders with nanosize structure.

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